## LOYOLA COLLEGE (AUTONOMOUS), CHENNAI - 600034

## M.C.A. DEGREE EXAMINATION - COMPUTER APPLICATIONS

FIRST SEMESTER - NOVEMBER 2015
CA 1804 - DISCRETE STRUCTURES

Date: 03/11/2015
Time : 01:00-04:00
PART A

## Answer ALL Questions

(10 X 2 = 20 Marks $)$

1. Define proposition in mathematical logic.
2. What is Principle Conjunctive Normal Form?
3. Define relation.
4. How will you define a lattice?
5. Define function.
6. State pigeonhole principle.
7. What is bipartite graph? Give example.
8. A connected graph contains Eular circuit iff $\qquad$ of its vertices is of $\qquad$ degree.
9. What is called group?
10. What is group homomorphism?

## PART B

## Answer ALL Questions

11a. Construct truth table for the following compound proposition:

$$
(\mathbf{p} \rightarrow \mathbf{q}) \wedge(\mathbf{q} \rightarrow \mathbf{r}) \rightarrow(\mathbf{p} \rightarrow \mathbf{r})
$$

(or)
11b. Prove the following equivalences by proving the equivalence of the duals.
$\neg\left(\left(\neg \mathbf{p}^{\wedge} \mathbf{q}\right) \mathbf{v}\left(\neg \mathbf{p}^{\wedge} \neg \mathbf{q}\right)\right) \mathbf{v}\left(\mathbf{p}^{\wedge} \mathbf{q}\right) \equiv \mathbf{p}$
12a. i Define onto function.
ii. Determine whether the following functions are one-to-one, onto, or one-to-one \& onto
(a) $\mathrm{f}: \mathrm{Z} \longrightarrow \mathrm{Z}$ defined by $\mathrm{f}(\mathrm{x})=\mathrm{x}^{2}-8 \mathrm{x}+15$ (b) $\mathrm{f}: \mathrm{Z} \longrightarrow \mathrm{Z}$ defined by $\mathrm{f}(\mathrm{x})=\mathrm{x}+12$
(or)
12b. If $f(x)=x+2, g(x)=x-2$ and $h(x)=3 x$ for $x \in R$, where $R$ is the set of real numbers;
Prove that i. $f \bullet g=g \bullet f$ ii. $g \bullet h \neq h \bullet g$ iii. $h \bullet f \neq f \bullet h$ iv. $(f \bullet g) \cdot h=f \bullet(g \bullet h)$
13a. i. In how many ways can 6 boys and 4 girls sit in a row?
ii. In how many ways can they sit in a row if boys are to sit together and girls sit together
iii. In how many ways can they in a row if girls to sit together?
iv. In how many ways can they sit in a row if just the girls to sit together?
(or)
13b. 7 women and 9 men are on the faculty in the computer science department of a college. How many ways are there to select a committee of 5 members
i. if at least one woman must be on the committee
ii. if at most two men must be on the committee

14a. i. What is adjacency matrix?
ii. Give adjacency matrix for (a) rectangle having two diagonals (b) triangle.
(or)
14b. Define the following with suitable examples
i. Euler graph ii. Connected graph iii. Planar graph iv. Complete graph

15a. i. Define monoid .
ii If $Z_{6}=\{0,1,2,3,4,5\}$ and $X_{6}$ is multiplicative modulo 6 then show that $\left(Z_{6}, X_{6}\right)$ forms a monoid. Also show that it is commutative.
(or)
15b. Prove that the necessary and sufficient condition for a non-empty subset H of a group $(G, *)$ to be a sub group is $a, b, \quad H=>b^{-1} \quad H$

## PART C

## Answer any TWO Questions

16a. i. Verify De Morgan's law using truth table
ii. Without constructing the truth table, find the principal disjunctive normal form of the following $(\mathbf{p} \boldsymbol{\wedge} \mathbf{q}) \mathbf{v}(\neg \mathbf{p} \boldsymbol{\wedge} \mathbf{q}) \mathbf{v}(\mathbf{q} \mathbf{\wedge} \mathbf{r})$

16b. i. Give procedure to draw Hasse diagram?
ii Let $\mathrm{A}=\{1,2,3,4,6,12\}$ and R is defined by aRb if a exactly divides b. Draw Hasse and find the greatest and least elements.

17a. There are 250 students in a college of which 188 can speak Tamil, 100 can speak Hindi, 35 can speak telugu. Further 88 can speak both Tamil and Hindi, 23 can speak Tamil and Telugu and 29 can speak Hindi and Telugu. If 19 can speak all the three languages then how many of these 250 can not speak any of the three languages.

17b. Using Dijkstra algorithm, find the shortest path between vertex $A$ and vertex $F$ in the following graph.


18a. i. If $(G, *)$ is abelian group show that $(a * b)^{n}=a^{n} * b^{n}$
ii. Show that $(E,+)$ is a subgroup of $(Z,+)$, where $E$ is set of even integers and $Z$ is set of integers.
18b. i. Show that the set $\mathrm{Q}^{+}$(set of positive rational numbers) forms an abelian group under the * operation which is defined by $a * b=a b / 2$ where $a, b \in Q^{+}$.
ii. If $\mathrm{Z}_{5}=\{0,1,2,3,4\}$ and $*$ is defined by addition modulo 5 then show that it is a group.

